

## Problem 1.33

[Difficulty: 2]

**1.33** The maximum theoretical flow rate (slug/s) through a supersonic nozzle is

$$\dot{m}_{\max} = 2.38 \frac{A_t p_0}{\sqrt{T_0}}$$

where  $A_t$  (ft<sup>2</sup>) is the nozzle throat area,  $p_0$  (psi) is the tank pressure, and  $T_0$  (°R) is the tank temperature. Is this equation dimensionally correct? If not, find the units of the 2.38 term. Write the equivalent equation in SI units.

**Given:** Equation for maximum flow rate.

**Find:** Whether it is dimensionally correct. If not, find units of 2.38 coefficient. Write a SI version of the equation

**Solution:** Rearrange equation to check units of 0.04 term. Then use conversions from Table G.2 or other sources (e.g., Google)

"Solving" the equation for the constant 2.38:

$$2.38 = \frac{\dot{m}_{\max} \sqrt{T_0}}{A_t \cdot p_0}$$

Substituting the units of the terms on the right, the units of the constant are

$$\frac{\text{slug}}{\text{s}} \times \text{R}^{\frac{1}{2}} \times \frac{1}{\text{ft}^2} \times \frac{1}{\text{psi}} = \frac{\text{slug}}{\text{s}} \times \text{R}^{\frac{1}{2}} \times \frac{1}{\text{ft}^2} \times \frac{\text{in}^2}{\text{lbf}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = \frac{\text{R}^{\frac{1}{2}} \cdot \text{in}^2 \cdot \text{s}}{\text{ft}^3}$$

$$c = 2.38 \frac{\text{R}^{\frac{1}{2}} \cdot \text{in}^2 \cdot \text{s}}{\text{ft}^3}$$

Hence the constant is actually

For BG units we could start with the equation and convert each term (e.g.,  $A_t$ ), and combine the result into a new constant, or simply convert  $c$  directly:

$$c = 2.38 \frac{\text{R}^{\frac{1}{2}} \cdot \text{in}^2 \cdot \text{s}}{\text{ft}^3} = 2.38 \frac{\text{R}^{\frac{1}{2}} \cdot \text{in}^2 \cdot \text{s}}{\text{ft}^3} \times \left( \frac{\text{K}}{1.8 \text{R}} \right)^{\frac{1}{2}} \times \left( \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \right)^2 \times \frac{1 \cdot \text{ft}}{0.3048 \text{m}}$$

$$c = 0.04 \cdot \frac{\text{K}^{\frac{1}{2}} \cdot \text{s}}{\text{m}} \quad \text{so} \quad \dot{m}_{\max} = 0.04 \frac{A_t \cdot p_0}{\sqrt{T_0}} \quad \text{with } A_t \text{ in m}^2, p_0 \text{ in Pa, and } T_0 \text{ in K.}$$